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III. Wave Model

Following is a description of the wave model used for the simulation.

Two main effects will transform waves arriving at Sardinera Bay: refraction due to the changing depth, and diffraction due to the presence of the breakwater(s). In order to accurately simulate these transformations, a mild slope model equation has been used, following Kirby (1994).

The equation may be written as:

$$C_g A_x + i(\bar{k} - k)C_g A + \frac{\omega}{2}\left(\frac{C_g}{\omega}\right)_x A - \frac{i}{2\omega}\left(CC_g A_y\right)_y + \frac{\omega}{2} A - \frac{\omega k^2}{2} D|A|^2 A = 0, \qquad (1)$$

where A is the wave amplitude, ω is the wave frequency, k is the wavenumber, and the dispersion relation is given by:

$$\omega^2 = gk \tanh(kh). \qquad (2)$$

Where h is the water depth and $g = 9.8\ m/s^2$ is the acceleration due to gravity. The phase velocity and the group velocity are defined as:

$$C = \frac{\omega}{k} \qquad (3)$$

and

$$C_g = \frac{\partial \omega}{\partial k}, \qquad (4)$$

respectively.

From the Coastal Engineering Manual, (Part II, Section 3 Page 25): "Wave nonlinearity has a strong effect on the phase speed of waves and thus can significantly modify both refraction and diffraction effects. For example, waves shoaling on a plane beach refract more slowly than predicted by linear theory, since the increase in wave-height with decreasing water depth speeds up the waves, in opposition to the direct, linear-theory effect of decreasing depth, which slows them. Diffraction effects are typically amplified. Phase speed is greater in a high-amplitude illuminated area than in a low-amplitude, shadowed area; this causes refractive bending of waves into the shadow area, causing an increase in wave height in the shadow zone relative to the predictions of linear theory."

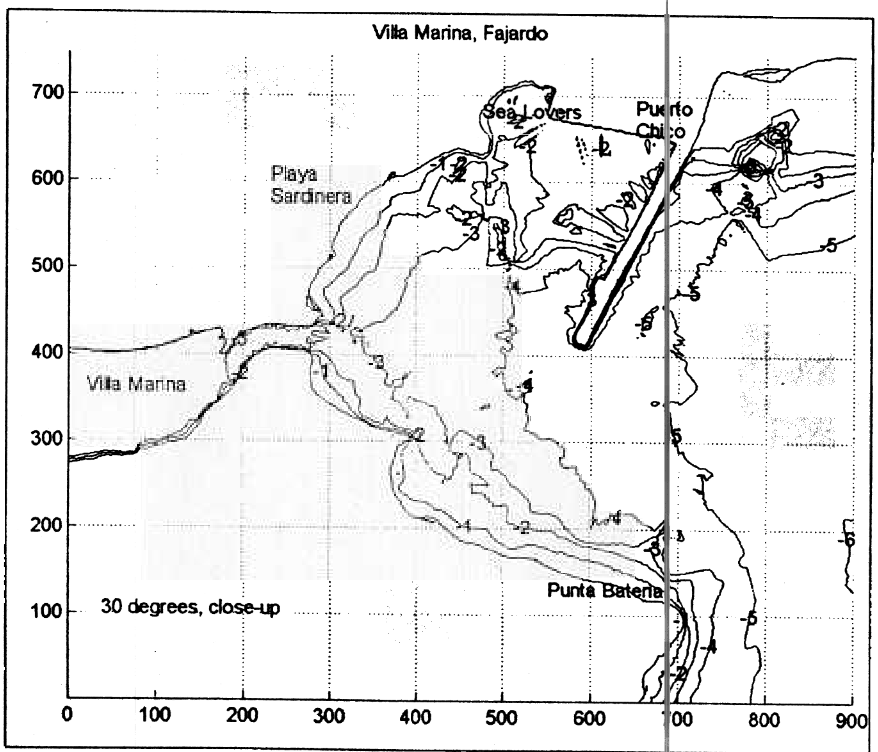

Figure 2. Computational Grid oriented at 30 degrees, close-up ve

The wave model equation (1) used in the simulation, designed to predict the propagation of a monochromatic wave in intermediate water depth, includes the effects of nonlinearity as predicted by third-order Stokes wave theory. Comparison of model results with laboratory data by a number of investigators including Berkhoff, Booij and Radder (1982), Tsay and Liu (1982), Kirby and Dalrymple (1984), Panchang. *Et al* (1990), and Demirbilek (1994) has shown that the higher-order parabolic approximation, together with nonlinear correction to the wave phase speed, can correctly predict the distribution of wave heights and nodal points in the evolving wave field.

A computational grid must be specified, and the model requires the bathymetry at each point on the grid as input. The estimation of the bathymetry is described in *Bathymetric Analysis for Villa Marina, Fajardo*. The resolution of the grid is chosen so as to satisfy stability criteria involving the speed with which information is propagated throughout the domain. In this case a grid with a 2-meter resolution was employed.

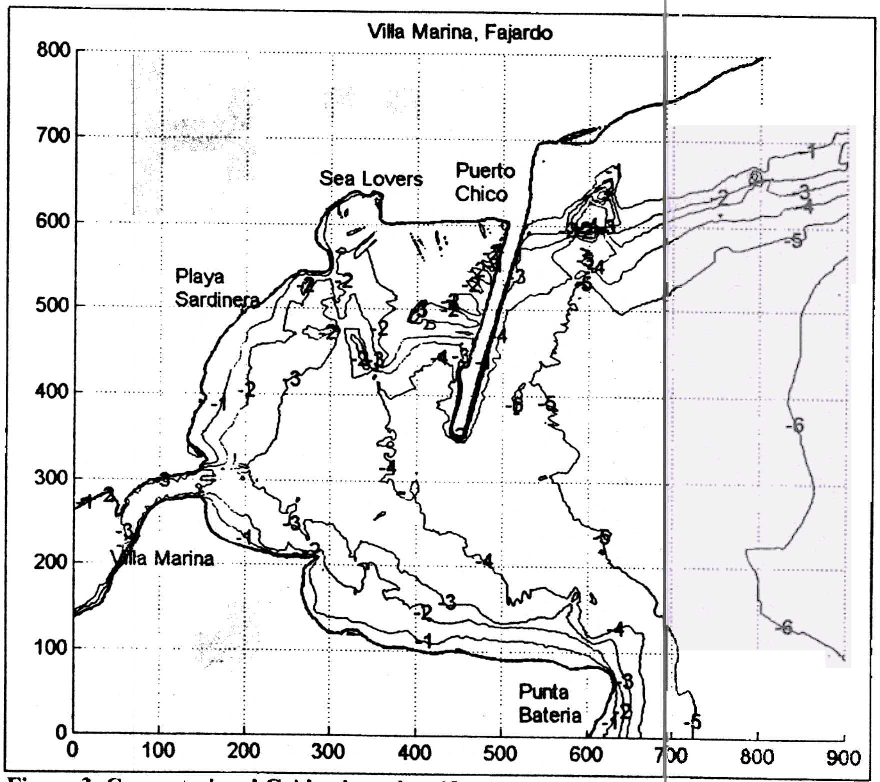

Figure 3. Computational Grid oriented at 18 degrees, close-up v

The period and the complex amplitude of the wave are specified at one of the boundaries of the computational grid, and propagated row-by-row using an implicit Crank-Nicholson procedure involving the inversion of a tri-diagonal matrix with complex-valued coefficients.

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Because of this, the computational grid must be oriented to coincide with the desired incident wave direction. In this case two computational grid orientations were chosen (18° and 30°) to correspond to the two exposed directions at Sardinera Bay (See Section II. Wave Climate; See Figures 4 and 5).

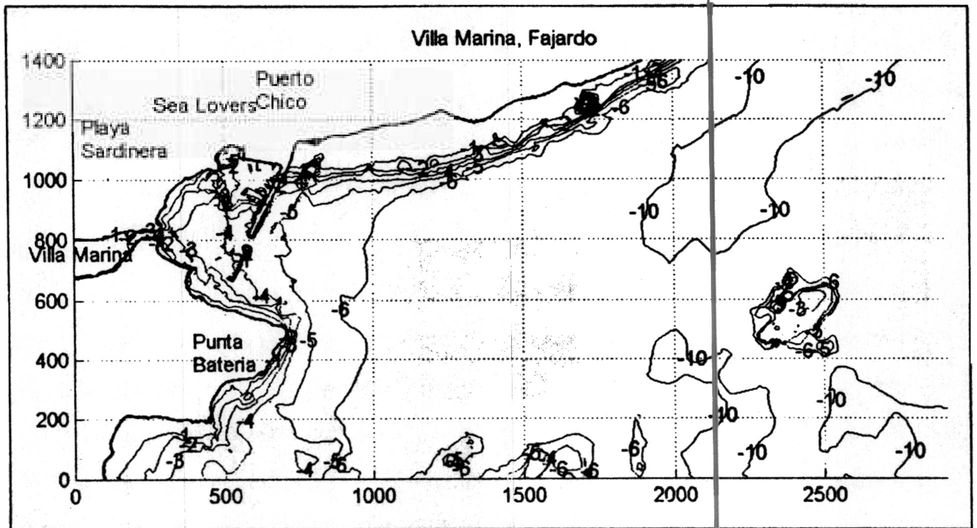

Figure 4. Computational Grid oriented at 30 degrees. full version.

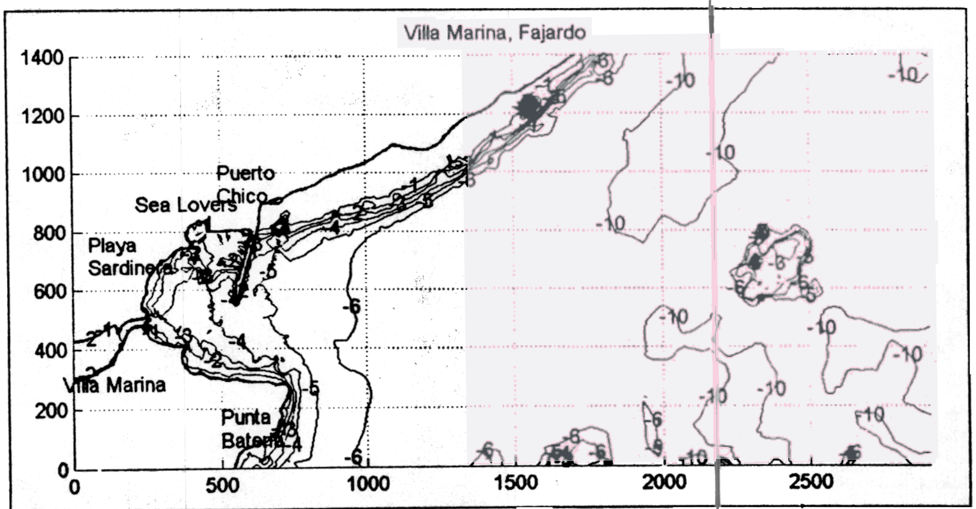

Figure 5. Computational Grid oriented at 18 degrees, full version.

In addition, two domain sizes were chosen for each direction. In order to characterize the wave transformations caused by the approach to the bay, a grid 2900 meters long and 1400 meters wide was used. In order to examine the effects of the bathymetry within Sardinera Bay itself on the incoming waves, a smaller grid 900 meters long and 800 meters wide was used. (See Figures 2-5.)

Furthermore, the above-mentioned wave conditions were simulated using four additional domains, corresponding to the bathymetries discussed above, modified by the presence of a breakwater as proposed by the owners of Villa Marina Yacht Harbor. (See Figure 6.)

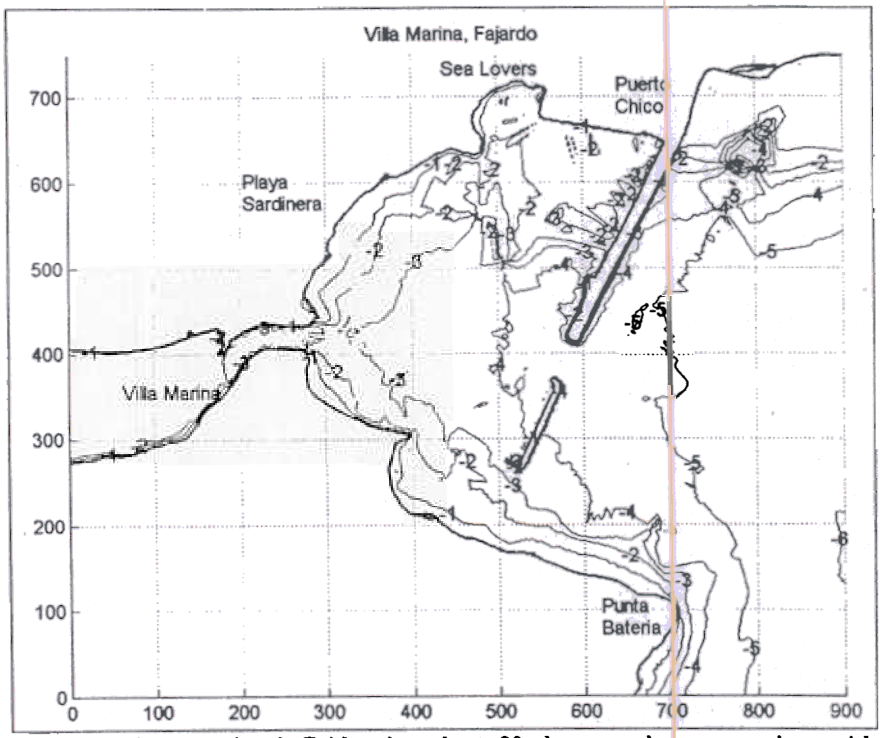

Figure 6. Computational Grid oriented at 30 degrees, close-up version, with proposed breakwater for Villa Marina Yacht Harbor included.

The output from the model is the normalized complex amplitude of the resulting wave heights at each point on the computational grid. In addition, a calculation of the normalized energy flux is made, which involves the product of the wave amplitude and the square root of the group velocity. These results are described in the next section.

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IV. Results

For description purposes, the waves have been divided into three categories. These are:

a) Waves generated by trade winds: corresponding to every day conditions, with periods ranging from 3 to 6 seconds and amplitudes ranging from 0.5 to 1.5 meters. We will consider the results from the T = 5.0 seconds waves as representative of this category.

b) Waves generated by weak to moderate extra-tropical cyclones, and tropical cyclones up to category 2 hurricanes: corresponding to typical winter swell conditions, with periods ranging from 7 to 12 seconds and amplitudes ranging from 2 to 10 meters. We will consider the results from the T = 9.0 seconds waves as representative of this category.

c) Waves generated by moderate to intense extra-tropical cyclones, and tropical cyclones up to category 4 hurricanes: corresponding to hazardous or even catastrophic swell conditions, with periods ranging from 9 to 15 seconds and amplitudes ranging from 5 to 15 meters. We will consider the results from the T = 13.0 seconds waves as representative of this category.

The results of the refraction/diffraction analysis will be discussed in the following order:

1) The effects of the existing bathymetry on the wave field along the approach to Sardinera Bay for waves with periods of 5, 9 and 13 seconds, arriving through both the 18° and 30° windows, will be examined.

2) The effects of the bathymetry on the wave field within Sardinera Bay itself, as generated by uniform wave trains (unaffected by the bathymetry of the approach) arriving from the 18° and 30° directions, with periods of 5, 9 and 13 seconds, will be described.

3) The effects of the existing bathymetry on the wave field along the approach to Sardinera Bay for waves with periods of 5, 9 and 13 seconds, arriving through

both the 18° and 30° windows will be discussed, as relates to their impact on the proposed breakwater for Villa Marina Yacht Harbor.

4) The effects of the proposed breakwater for Villa Marina Yacht Harbor on the wave fields within Sardinera Bay itself, as generated by uniform wave trains (again unaffected by the bathymetry of the approach) arriving from 18° and 30°, with periods of 5, 9 and 13 seconds, will be described.

1) The Approach to Sardinera Bay

The bathymetry along the approach to Sardinera Bay can be characterized in general as a monotonically, uniformly varying bottom. It ranges in depth from about ten (10.0) meters at the easternmost end of the computational grid, to some five (5.0) meters at the mouth of the Bay itself, approximately two (2.0) kilometers to the west.

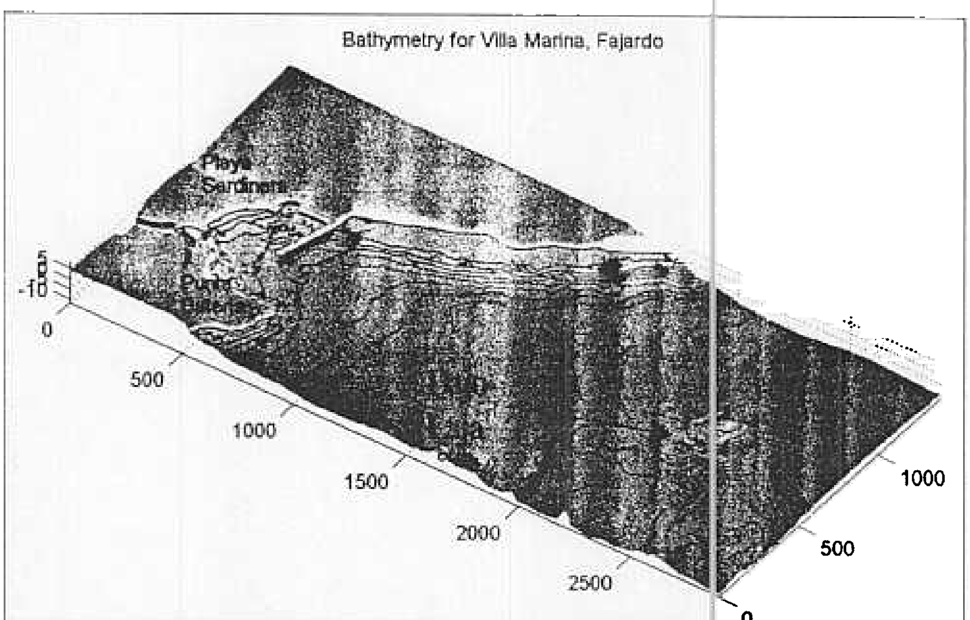

Figure 7 Three dimensional representation of bathymetry for approach to Sardinera Bay, Fajardo, with Bajo Laja visible in the foreground.

There is, however, one noteworthy feature to the bathymetry of the approach to Sardinera Bay: a small seamount known as "Bajo Laja". This seamount is located almost halfway between Isleta Marina and the El Conquistador Hotel, at latitude 18° 21.129' North, longitude 65° 37.024' West. The depths at Bajo Laja can be as shallow as three meters, and the entire formation is about two hundred (200.0) meters in diameter (See Figure 7).

Bajo Laja is an extremely important feature because the energy of the waves that are refracted as they pass over Bajo Laja gets focused into beams, a situation which can lead to dramatic increases in the energy levels at specific locations. In effect, Bajo Laja acts as a lens, focusing the incoming wave energy into relatively narrow beams.

This "lens effect" is dominant at all periods and directions, visible as a "hole" in the power plots directly over Bajo Laja, accompanied by a series of beams emanating from this location towards Sardinera Bay.

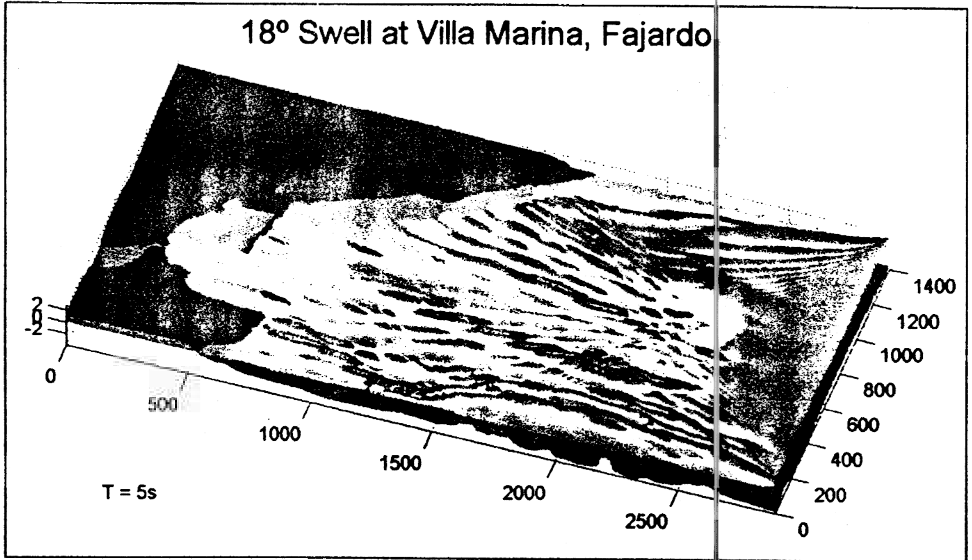

Figure 8. Wave power arriving at Sardinera Bay, Fajardo (Direction = 18°, Period = 5s).

The waves with a period of five seconds (T = 5s), which correspond to everyday waves generated by easterly trade winds, and have amplitudes typically ranging from 0.5 to 1.5 meters, are almost entirely focused into the central and northern portions of the Puerto Chico breakwater, when they arrive through the 18° window between Cayo Lobos and Isla Palominos (See Figure 8.). Therefore we can conclude that the wind waves arriving through the 18° window have no significant impact on Sardinera Bay.

In contrast, when arriving through the 30° window, in between Cayo Icacos and Cayo Lobo, the five-second period energy is focused further south. A beam of wave energy impacts the southernmost end of the Puerto Chico breakwater. In addition, two other beams penetrate Sardinera Bay some 30 meters and 150 meters south of the breakwater tip (See Figure 9). It is believed that these are the waves responsible for the daily chop within Sardinera Bay.

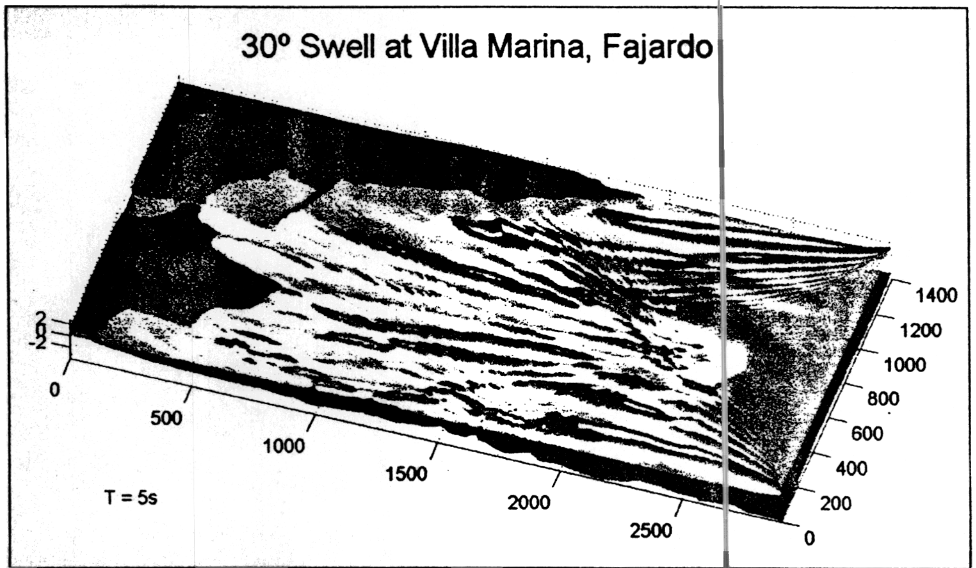

Figure 9. Wave power arriving at Sardinera Bay, Fajardo (Direction = 30°; Period = 5s).

When it comes to swell resulting from moderate extra-tropical cyclones or hurricanes up to category 2 (represented by the T = 9s wave), if it arrives via the 30° window the impact is largely concentrated on the southern tip of Puerto Chico's breakwater. If the swell arrives through the 18° window, its energy impacts the southern tip of the breakwater as well, but there is in addition a significant amount of energy transferred into Sardinera Bay itself by means of a second beam (See Figure 10). These waves typically range in amplitude from 2 to 10 meters, and can be expected to impact Sardinera Bay 10 to 12 times a year.

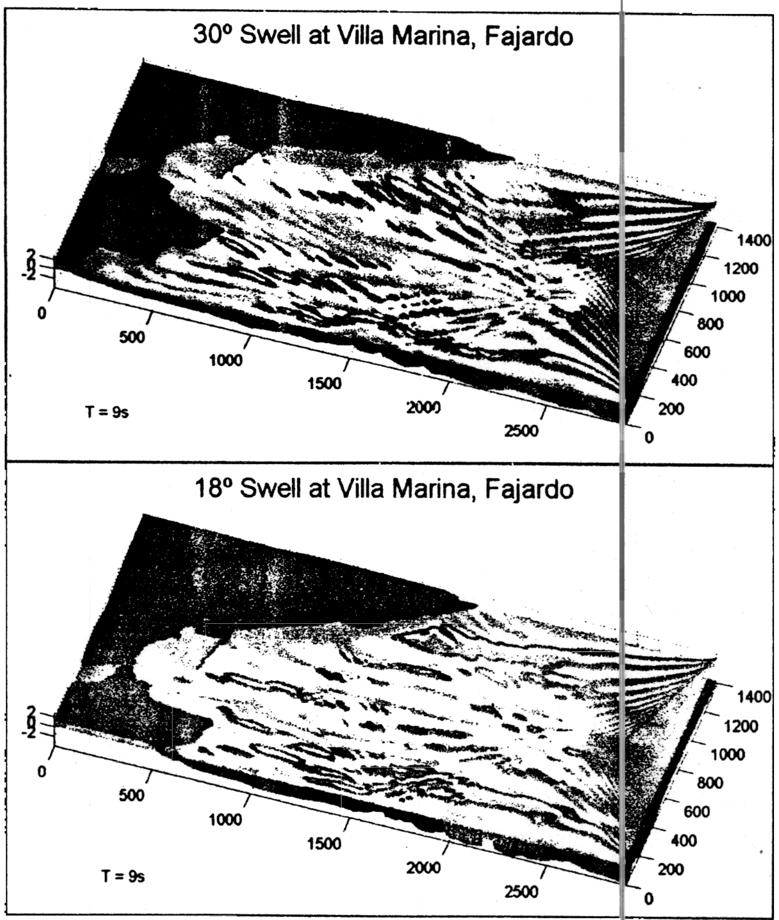

Figure 10. Wave power arriving at Sardinera Bay, Fajard 18°, 30°; Period = 9s).

Finally, examination of the behavior of waves generated by intense extra-tropical cyclones or hurricanes up to category 4 (represented by the T = 13s wave), reveals that the situation is similar to that of the waves previously discussed, only with the incoming directions reversed. That is, if the swell arrives via the 18° window the impact is largely concentrated on the southern tip of Puerto Chico's breakwater. If the swell arrives through the 30° window, its energy impacts the southern tip of the breakwater as well, but there is in addition a significant amount of energy transferred into Sardinera Bay itself by a second means of a second beam (See Figure 11). These catastrophic events can generate wave heights of up to 15 meters, and should not be expected more often than once a decade or so.

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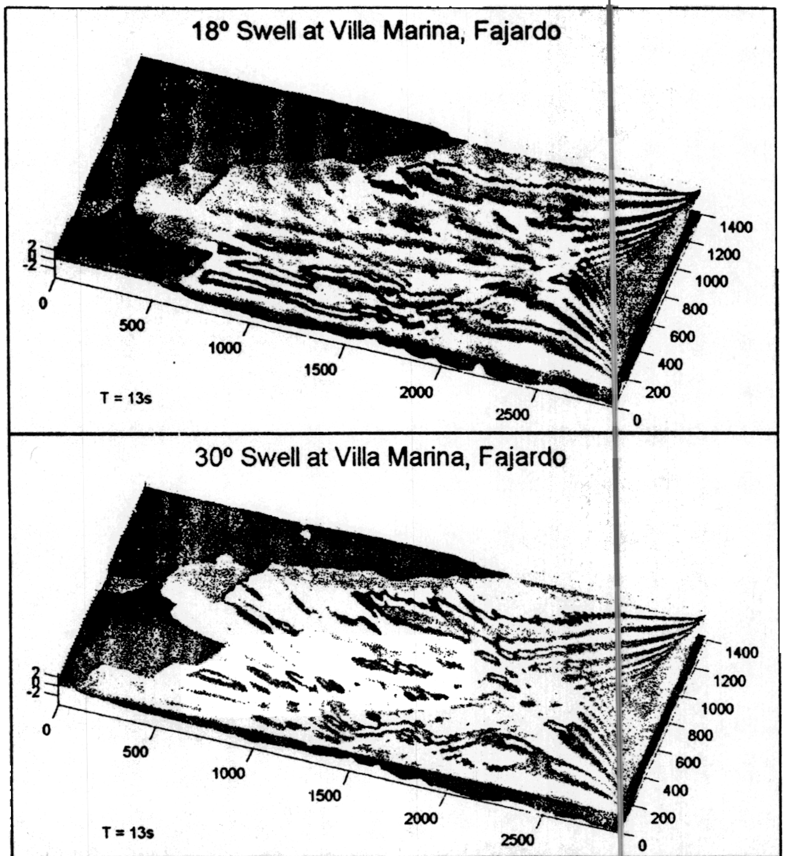

Figure 11. Wave power arriving at Sardinera Bay, Fajardo (Direction = 18°, 30°; Period = 13s).

It is believed that a combination of wave energy beams resulting from waves similar in nature to the T = 9s and T = 13s period waves simulated here, arriving through one or both of the windows discussed above, are responsible for the destruction of the southern tip of Puerto Chico's breakwater during Hurricane Georges in 1998. A more graphic example of the consequences of the focusing of wave energy cannot be made.

2) Sardinera Bay

Within Sardinera bay itself, again we find a relatively gently sloping bottom which ranges from five (5.0) meters at the mouth to zero (0.0) meters at the shoreline, approximately five hundred (500.0) meters to the west.

Once again, there is a single noteworthy feature in the bathymetry of Sardinera Bay: the breakwater that extends some two hundred and fifty (250.0) meters southward from the northern end of Sardinera Bay, and protects the Puerto Chico Marina. This breakwater is important not only because of the protection it affords the boats in Puerto Chico Marina, but because of the refractive effects its presence has on the wave field within Sardinera Bay (See Figure 12).

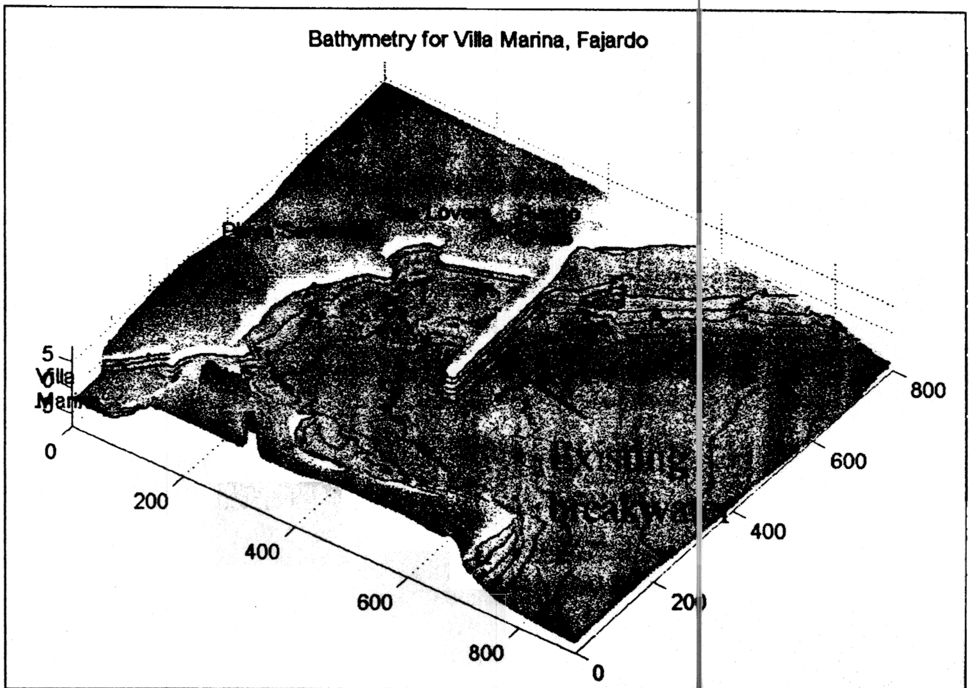

Figure 12. Three dimensional representation of bathymetry for Sardinera Bay, Fajardo, with Puerto Chico breakwater visible in the foreground.

If one were not to take into account the effects of wave diffraction, the breakwater protecting the north side of Sardinera Bay would effectively slice any incoming plane in half, allowing the wave to proceed unaffected through the southern half of the bay, while keeping the northern half in its shadow. If this were the case, all boats in both Sea Lovers Marina and Puerto Chico Marina would be equally and completely sheltered from any incoming wave energy.

In reality, whenever a wave is forced through a gap, or blocked by an object such as an island or a breakwater (which is equivalent to half a gap), the phenomenon known as diffraction occurs. The end result is that the shadow produced by the breakwater is not a perfect one, as envisioned above. In fact the wave energy penetrates the "shadow zone" much the way the waves from a stone dropped at the end of the breakwater would (where the wave is cut in two).

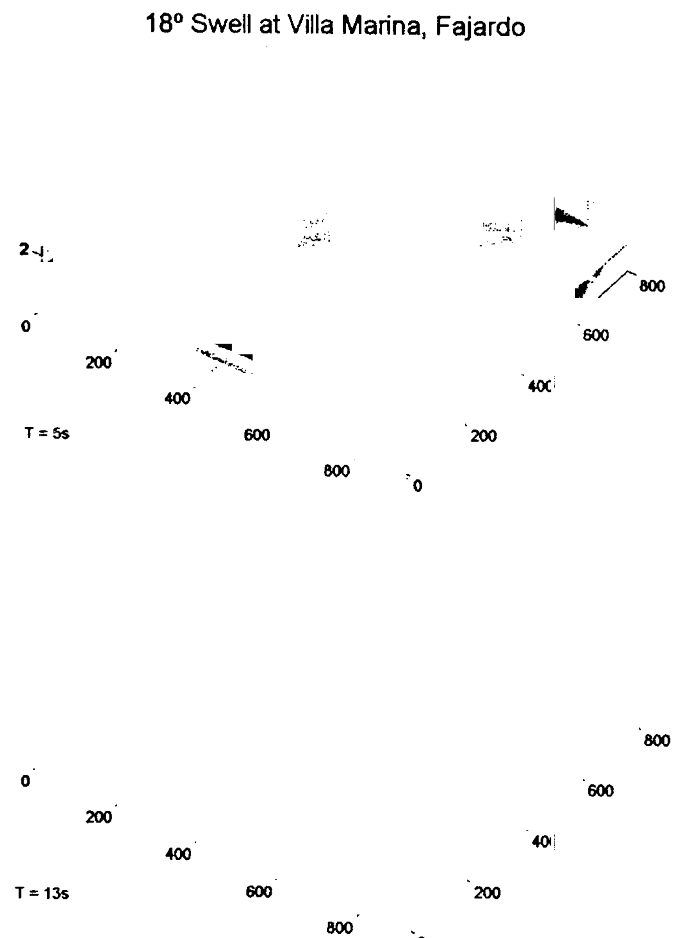

Figure 13. Waves refracted by the Puerto Chico breakwater in Sardinera Bay, Fajardo. (Direction = 18°; Period = 5s, 13s).

This effect is visible in the refraction/diffraction diagrams for all periods (T = 5, 9, 13 seconds) and all directions (18° and 30°). As theory predicts, diffraction is stronger for longer wavelengths (and thus longer periods). As intuition predicts, the penetration of diffracted wave energy is greater for the 18° waves than for the 30° waves, since they impact Sardinera Bay at a more direct angle (See Figure 13).

It is clear that wave diffraction is an important phenomenon in determining the wave field within Sardinera Bay. It is this phenomenon that is responsible for the "choppiness" during everyday conditions and the rolling motion during storm conditions experienced in Sea Lovers Marina and in the southern most slips on the western docks of Puerto Chico as compared to the conditions in the rest of Puerto Chico Marina and within neighboring Villa Marina.

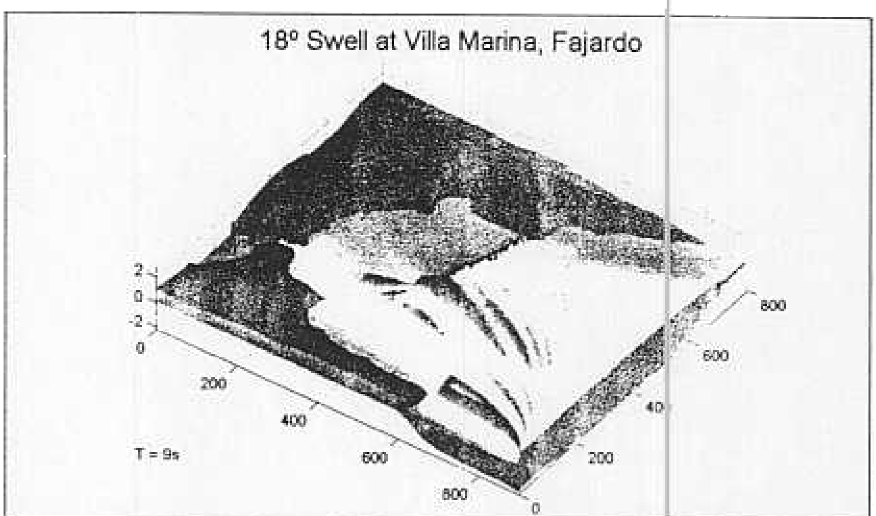

Figure 14. Wave power impacting Sardinera Bay, Fajardo (Direction = 18°; Period = 9s).

The power diagrams for all three-wave periods and both directions show the bulk of the energy concentrating at the tip of the breakwater and just south of it. The energy penetrates deep into the southern part of Sardinera Bay, dissipating between the entrance to Villa Marina Yacht harbor and the small outcropping just to the southeast. Perhaps a bit more so than anticipated, the energy from the 18° window tends to be more tightly packed near the tip of the breakwater. (see Figures 14-15).

Clearly the combination of this natural tendency of the bay itself, combined with the focusing effect of Bajo Laja described earlier make the probability of some very localized and intense waves impacting at or near the tip of the breakwater rather high.

The need for adequate fortification of the breakwater at its tip, as well as along its entire length is clear.

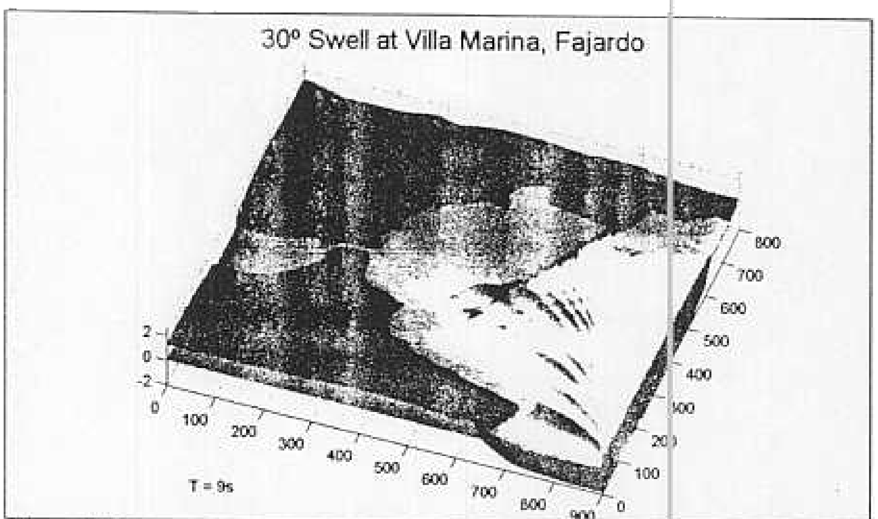

Figure 15. Wave power impacting Sardinera Bay, Fajardo (Direction = 30°; Period = 9s).

3) The Approach to Sardinera Bay and the proposed breakwater for Villa Marina

In this section, we examine the effects of the bathymetry of the approach to Sardinera bay upon waves of the same periods (T = 5, 9, 13 seconds) and directions (18° and 30°) as above, with respect to their impact upon a proposed breakwater for Villa Marina Yacht Harbor (See Figure 16).

All previous observations about the distribution and concentration of the wave energy in the approach to Sardinera Bay still apply, since the proposed breakwater does not influence incoming waves until they impinge upon it. (There is potential for some of the wave energy to be reflected causing changes in the nature of the wave field to the east of the proposed breakwater, but they are not contemplated in this analysis.) Next, we will examine the portions of the wave energy that directly impact the proposed breakwater.

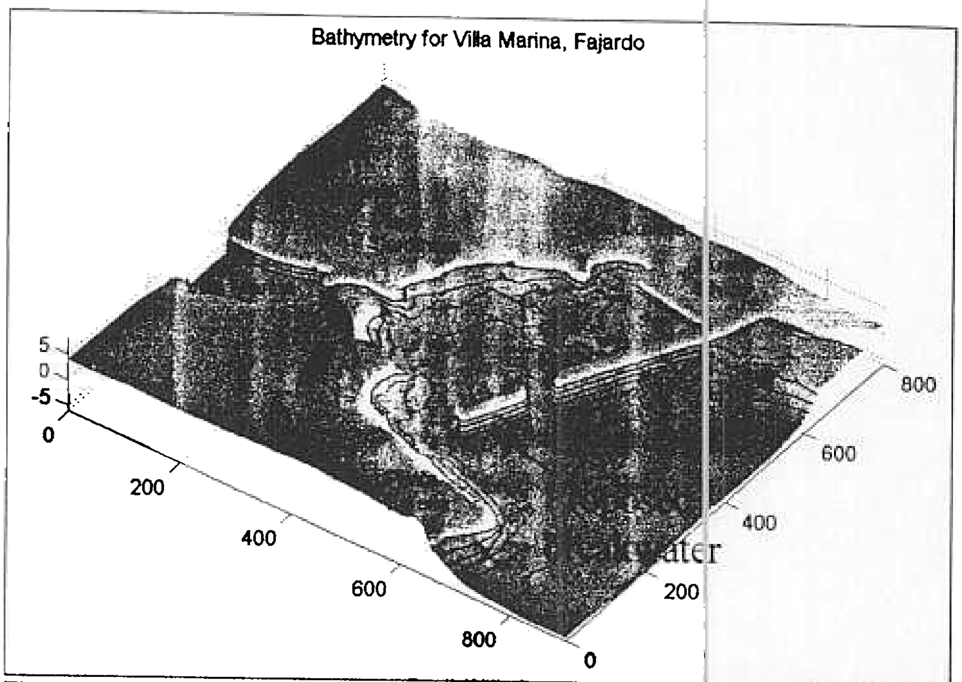

Figure 16. Three dimensional representation of bathymetry for Sardinera Bay, Fajardo, with proposed breakwater for Villa Marina Yacht Harbor visible in the foreground.

The proposed breakwater for Villa Marina will receive relatively little energy from every day wind swell (modeled in this analysis by the T = 5s wave). As discussed above, the energy from these waves impacts mainly the Puerto Chico breakwater when arriving through the 18° window. When arriving through the 30° window, the wave energy is concentrated just south of the Puerto Chico breakwater, in between it and the proposed breakwater. There is a second beam of energy just south of the proposed breakwater (See Figure 17).

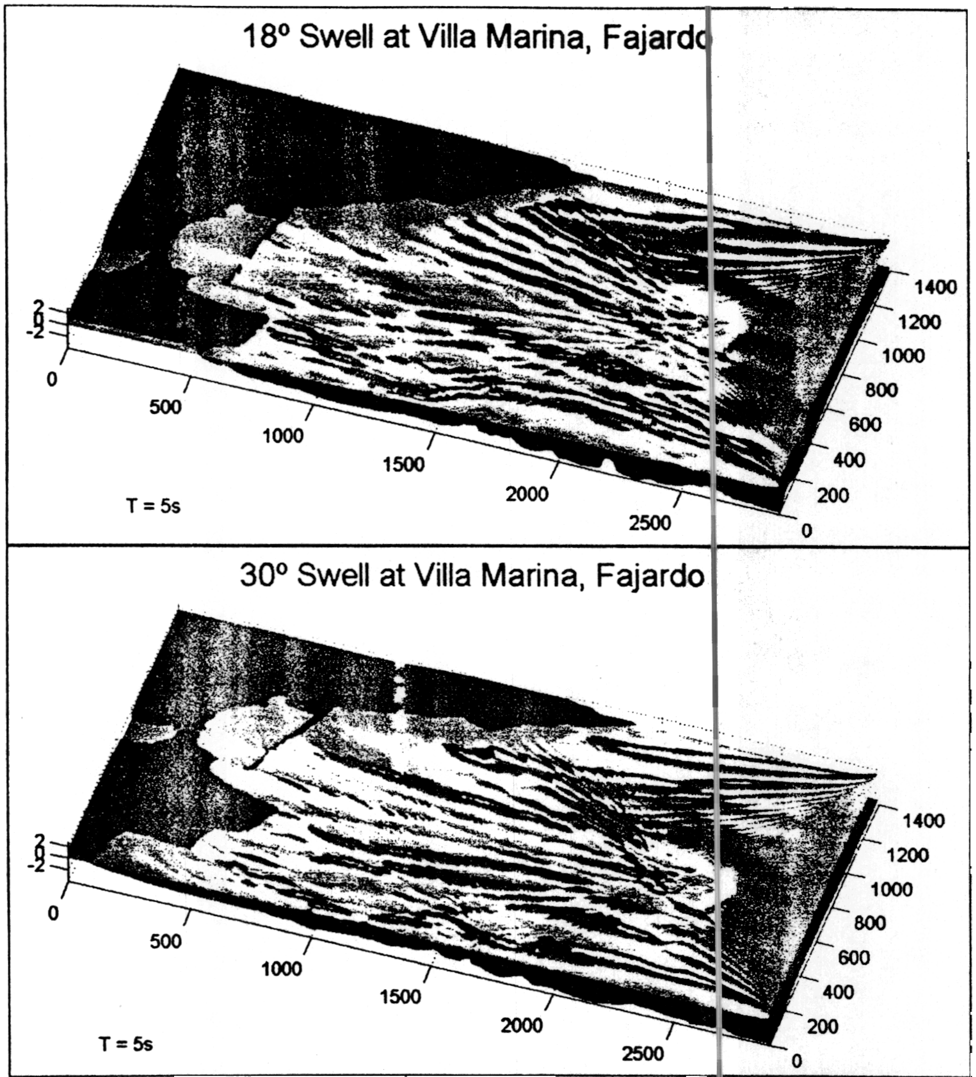

Figure 17. Wave power arriving at Sardinera Bay, Fajardo, with the proposed breakwater for Villa Marina Yacht Harbor included (Direction = 18°, 30°; Period = 5s).

These every day wind waves typically have relatively small wave heights, between 0.5 and 1.5 meters. As such, despite the fact that they occur precisely where the ship lanes would be if the proposed breakwater were in place, they do not represent a navigational hazard. Rather, from the point of view of a ship captain, the impact of these waves should be considered as annoying but harmless.

In contrast, waves corresponding to moderate extra-tropical cyclones and hurricanes up to category 2 (modeled in this analysis by the T = 5s wave), will directly impact the proposed breakwater, when arriving through the 18° window (See Figure 18). As discussed above, when arriving through the 30° window, the energy from these types of waves is directed principally towards the Puerto Chico Breakwater. These waves typically range in amplitude from 2 to 10 meters, and can be expected to impact Sardinera Bay 10 to 12 times a year.

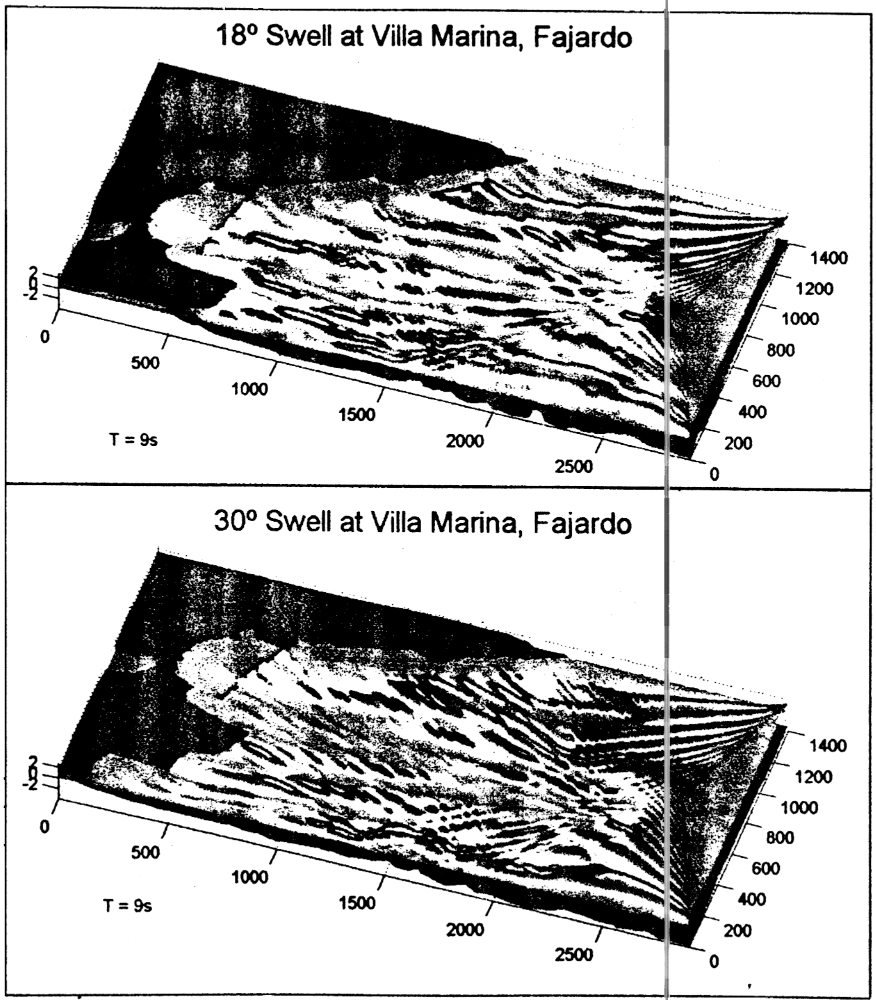

Figure 18. Wave power arriving at Sardinera Bay, Fajardo, with the proposed breakwater for Villa Marina Yacht Harbor included (Direction = 18°, 30°; Period = 9s).

Finally, waves of the type generated by severe extra-tropical cyclones and hurricanes of up to category 2 (modeled in this analysis by the T = 13s wave), will also impact the proposed breakwater, when arriving through the 30° window. As discussed above, when arriving through the 18° window, the energy from these types of waves is directed principally just south of the Puerto Chico Breakwater, and north of the location of the proposed breakwater for Villa Marina Yacht Harbor (See Figure 19). These catastrophic events can generate wave heights of up to 15 meters, and should not be expected more often than once a decade or so.

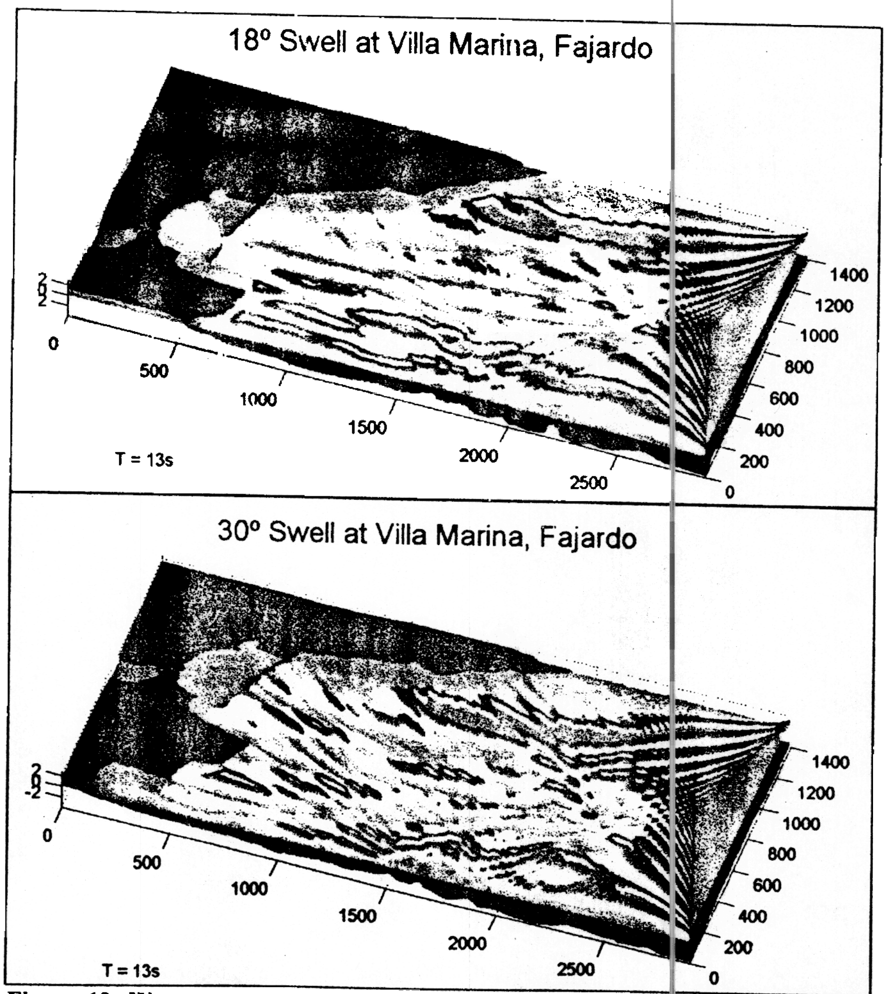

Figure 19. Wave power arriving at Sardinera Bay, Fajardo, with the proposed breakwater for Villa Marina Yacht Harbor included (Direction = 18°, 30°; Period = 13s).

It is evident that the proposed breakwater will be impacted by waves of significant energy from both available windows (18° and 30°). In order for the proposed breakwater to accomplish its purpose, to shelter the ships in its lee from the energy of incoming waves, it must be sufficiently fortified to be able to absorb the impact of these waves without being destroyed.

4) Sardinera Bay and the proposed breakwater for Villa M

The purpose of the proposed breakwater for Villa Marina Yacht Harbor is to shelter the ships in its lee from the energy of incoming waves. In this section we will discuss the ability of the proposed structure to do just that, in terms of its potential effect on the wave field within Sardinera Bay itself.

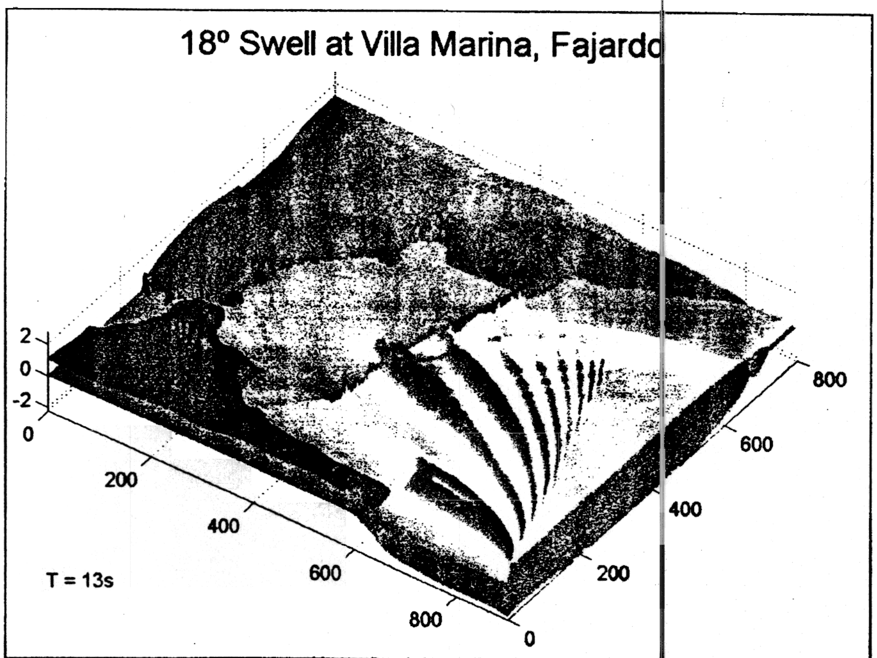

Figure 20. Power of waves generated by severe storms or hurricanes of up to category 4 arriving at Sardinera Bay, Fajardo, with the proposed breakwater for Villa Marina Yacht Harbor included (Direction = 18°; Period = 13s).

As stated above, the proposed breakwater for Villa Marina Yacht Harbor can be expected to be impacted directly by waves should storm or hurricane conditions arise. Provided the proposed breakwater is sufficiently strong, the refraction/diffraction diagrams show that adequate protection will be afforded to vessels located in its lee. (See Figure 20)

The wave energy that is focused into the gap in between the breakwaters by the bathymetry of the approach to Sardinera Bay from every type and direction of wave discussed above will be affected by wave diffraction. This is a fundamental aspect of any wave field that interacts with a "gap". In this case the wave energy penetrates the "shadow zone" much the way the waves from a stone dropped at each end of the gap would. This effect is dramatically visible in the wave amplitude figures in that the shape of the wave changes from a plane wave prior to entering the gap (crest is a straight line) to a spherical wave (crest is circular) after passing through the gap. The diffraction effect is a function of the period (or wavelength) of the incoming wave, and is greater for longer waves (See Figure 21).

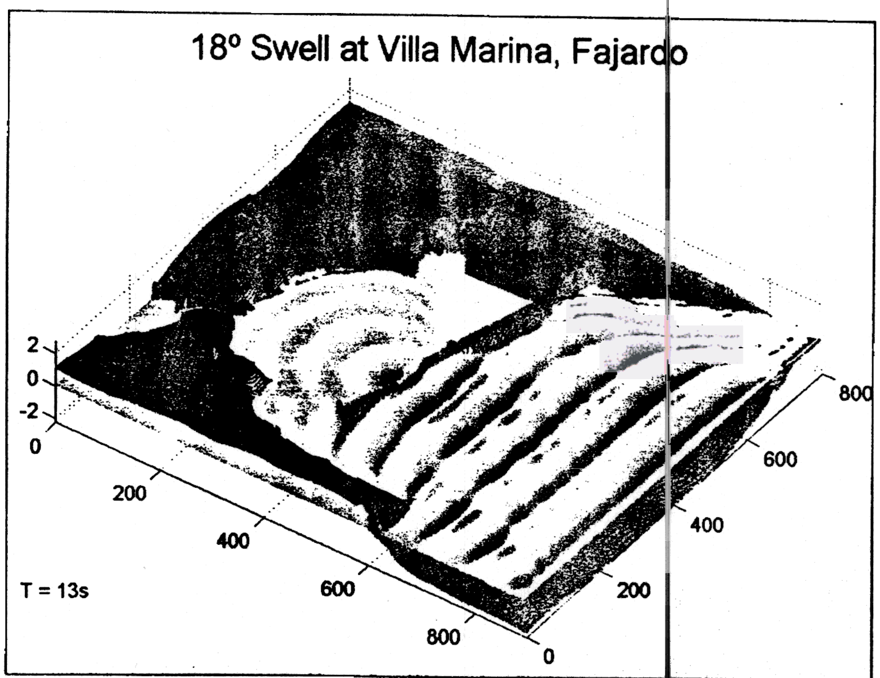

Figure 21. Amplitude of waves arriving at Sardinera Bay, Fajardo, with the proposed breakwater for Villa Marina Yacht Harbor included (Direction = 18°; Period = 13s).

The net result of the diffraction of the wave energy from the gap is the same as that from Puerto Chico's breakwater: wave energy "leaks" into the shadow zone behind the breakwater. The main difference is that now there are two shadow zones (one behind each breakwater) and that the energy diffracts in both directions (north and south), as opposed to only one (north). It is anticipated that the energy levels at the locations of the proposed expansion to Villa Marina Yacht Harbor will be comparable to those felt at Sea Lovers Marina and at the southernmost slips of the two westernmost docks at Puerto Chico Marina.

V. Summary

In summary, we have found that the bathymetry along the approach to Sardinera Bay refracts the incoming wave energy in such a way that it tends to be focused into narrow beams. These energy beams can be quite destructive in nature, and adequate construction parameters should be used when designing the proposed breakwater so as to insure its survivability in the face of such stresses as these energy beams may produce.

In addition we have found that, assuming the proposed breakwater maintains its structural integrity, the primary source for wave energy affecting the vessels in its lee will be due to wave diffraction. As such, the conditions at the site of the proposed expansion to Villa Marina yacht Harbor will be comparable to those found today at Sea lovers Marina.

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